How Fast Time Passes

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It takes some sophistication to become confused about space. It helps to be acquainted with non-Euclidean geometry, for instance, to get the bite of Henri Poincaré's argument that the geometry of space is chosen for convenience rather than discovered by investigation.[2] But it is dead easy to become confused about time.

Time, for instance, seems to pass, lapse, flow, fly, march, or just dribble away. Nevertheless, as soon as one tries to say literally what time is doing--the doing that distinguishes time from space--one can find oneself in familiar philosophical quagmires—infinite regresses, meaningless formulas, manifest contradictions.[3] But even if, as I think, one can emerge from these swamps with a relatively clear and cogent account of passage,[4] another problem inevitably arises.

Consider the opening paragraph of Arthur Prior's paper "Changes in Events and Changes in Things," which appeared in Prior (1968, 2003):

> The basic question to which I wish to address myself in this lecture is simply the old one, does time really flow or pass? The problem, of course, is that genuine flowing or passage is something which occurs *in* time, and *takes* time to occur. If time flows or passes, must there not be some 'super-time' in which it does so? Again, whatever flows or passes does so at some rate, but a rate of

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[1] I have been helped immensely by comments and criticisms from Richard Arthur and John Manchak, who may or may not agree with the results.

[2] See Poincaré (1952, Part II).

[3] The classic presentation of this lament is to be found in Williams (1951).

[4] Which I offer in Savitt (2002).

> flow is just the amount of movement in a given time, so how could there be a rate of flow of time itself? And if time does not flow at any rate, how can it flow at all?

It's this puzzle (or puzzle complex) that I wish to discuss in this paper. Let us take it as a working hypothesis that time does, somehow or other, pass and ask ourselves: How fast does time pass?

1. Rates and Regresses

When we wish to know how fast time passes, we seek a rate. Rates are typically ratios with the first item (or numerator) being the quantity whose rate of change is being sought and the second item (or denominator) being some difference in time. A ratio can indicate the rate of change of the first item per unit of time. For instance, if a vehicle travels 50 kilometers in 30 minutes, it travels at an average rate (or speed) of 100 kilometers per hour (100 km/h).

Rates of change themselves can change. If the speed of a car increases smoothly in one minute from 100 to 130 kilometers per hour, then the car accelerates at the average rate of 30 kilometers per hour per minute or 1/2 kilometer per hour per second (~.00014 km/sec^2).

But rates can be thought of more generally. The denominator of the rate need not be a time difference. For instance, the rate at which one pays income tax is the ratio of tax paid to taxable income. In a country with a progressive income tax the rate at which one pays tax (not just the amount of tax) increases with one's level of income.

If time flows or passes, it is natural[5] to suppose that it passes at some rate. We then have a problem when we try to find or to state this rate. Here, for instance, is Huw Price's statement of the problem (1996, 13):

[5] But not inevitable. Markosian (1993, 843) wonders briefly whether the supposition that there is a rate of the passage of time is a "category mistake." There may be something to this line of thought, but in this paper I will develop a different approach to the problem.

> [P]erhaps the strongest reason for denying the objectivity of the present is that it is so difficult to make sense of the notion of the objective flow or passage of time. Why? Well, the stock objection is that if it made sense to say that time flows, then it would make sense to ask how fast it flows, which doesn't seem to be a sensible question. Some people reply that time flows at one second per second, but even if we could live with the lack of other possibilities [that is, with there seeming to be no other possible rates at which time could "flow"], this answer misses the more basic aspect of the objection. A rate of seconds per second is not a rate at all in physical terms. It is a dimensionless quantity, rather than a rate of any sort. (We might as well say that the ratio of the circumference of a circle to its diameter flows at π seconds per second!)

Price seems to be thinking of the problem, familiar to physics students, of converting units. Suppose one were told, for instance, that a certain process took one fortnight, but one needed to know how long it took in seconds. One would convert units by "multiplying by 1" in the following way:

$$1 \text{ fortnight} = 1 \text{ fortnight } \times\left[\frac{2 \text{ weeks}}{\text{fortnight}}\right]\times\left[\frac{7 \text{ days}}{\text{week}}\right]\times\left[\frac{24 \text{ hrs}}{\text{day}}\right]\times\left[\frac{3600 \text{ s}}{\text{hour}}\right] = 1{,}209{,}600 \text{ s.}$$

The result is arrived at by multiplying the indicated numbers but canceling the corresponding units that appear in both numerator and denominator. But if units can be cancelled in this fashion, then in the apparent rate 1 second/second, canceling the units leaves one with just a "dimensionless quantity", or perhaps the pure number, 1. A number is not a rate.[6]

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[6] Nevertheless, Schutz (2009, 4) adopts a system of units in which the speed of light is just 1, a dimensionless quantity. The current (June, 2011) Wikipedia entry on Natural Units is a helpful introduction to the topic.

Another example, perhaps more apposite here, is that the accuracy of a watch or clock (Δt/T) is a dimensionless quantity that indicates the rate at which the watch runs. For instance, an accuracy of 1.15×10^{-5} indicates that a watch gains (to within some degree of error) one second per day. That is, it advances at a rate of 86,401 seconds per 86,400 seconds. (Lombardi, 2008)

One might try to avoid this apparent collapse of a rate into a pure number by supposing a second time dimension that differs from the first time dimension. If we indicate this second dimension by using capital letters, then it might be claimed that the rate of time's passage is 1 second per SECOND, and the units in this ratio do not cancel. The inevitable response to this claim is the observation that if the second time dimension is to be really a time dimension, then it too must flow or pass.[7] This flow or passage must in turn have a rate, necessitating the postulation of a third time dimension. It is easy to see that this process leads to an infinite hierarchy of time dimensions, an ontologically extravagant way to deal with the difficulty that was raised at the first level. So we find ourselves, with respect to time dimensions, in the situation that Nelson Goodman (1978, 119) thought we found ourselves in with respect to worlds:

> The philosopher like the philanderer is always finding himself stuck with none or too many.

A second problem sometimes raised is that genuine rates can have alternative values, but time could pass at no rate other than one second per second (or an equivalent supposed rate, like one hour per hour). Since there is no alternative rate for time's passing than one second per second, it is claimed that the alleged rate is not a real rate at all. So the initial "no rate" argument is buttressed by the secondary "no alternative rate" argument, as Michael Raven (2010) recently emphsized.

Of course, there are responses to these arguments. I would like to review a few of them before I propose my own.

The most striking reply is Tim Maudlin's argument that the rate of one second per second is, *pace* Price, a perfectly fine rate (Maudlin, 2007). Consider, he says, a fair exchange rate for currencies. Between currencies, a fair exchange rate might be defined in terms of purchasing power parity (PPP). Choose a basket of goods (The *Economist* magazine simplifies this to the limit by choosing only one item its basket, the ubiquitous

[7] One might try to deny that the second time dimension needs to be as dynamic as the first, but such a denial seems like special pleading. An extra time dimension, even if inert, is very puzzling, as I will insist below.

Big Mac, in its annual Big Mac index.), and consider their average price in local currencies. Then a fair exchange rate equalizes the price in the various currencies.

For example, in July, 2008 the average price of a Big Mac in the USA was $3.57 and in the UK it was £2.29. In this case, the implied fair exchange rate would be $3.57/£2.29 = 1.56 $/£. The actual exchange rate at that time was $2 to the pound, so this is a quick-and-dirty way to estimate that the pound was slightly overvalued in US$ (relative to this choice of basket of goods).

Similarly, Maudlin observes, the fair exchange rate of the US$ with respect to itself must be 1 $/$. "If you think that this answer is meaningless," Maudlin writes, "imagine your reaction to an offer of exchange at any other rate." (2007, 112) Since there can be only one fair exchange rate, the fact that there are no alternative fair exchange rates in no way detracts from the legitimacy of the 1 $/$ rate. So also the fact that time can pass at no rate other than 1 sec/sec should not detract from the legitimacy of that rate. Maudlin, it seems, answers both the "no rate" and the "no alternative rate" arguments.

Price (2011) has recently responded to Maudlin's arguments. He abandons the claim that the units in "one second per second" cancel, leaving a pure number.[8] He now insists that a ratio of a quantity to itself is no rate at all. Maudlin's example simply misses this central point, he says, since in his example one has a ratio of dollars offered to dollars returned. That ratio differs in an essential way from the ratio of 1 second to one second, or, for that matter, the ratio of one mile to one mile, which is merely the ratio of

[8] Olson (2009) still claims that the units cancel, leaving a pure number that can't be a rate. Skow (forthcoming a) claims that units cancel when one is simplifying within a system of units but not when one is converting from one system of units to another. He insists that pure numbers are to be distinguished from dimensionless quantities. The latter, but not the former, can indicate rates. In (forthcoming b, near the end of section 6) Skow offers an example different from Maudlin's in which units do not seem to cancel. Any reader who works through Skow's various formulations of the "moving spotlight" view, as a way of enabling one to say how fast time passes, would do well to consult also the less sanguine discussion of the moving NOW (currently section 9.4) in Meyer (forthcoming).

a quantity to itself. Neither is a rate, and the former is no more characteristic of time than the latter is of space.

Price does seem to have a point here. Nothing is more dynamic than time, but nothing seems less dynamic than the ratio of a quantity to itself. How could it be a rate, a measure of any sort of change, however broadly construed?

Does the no rate argument win, then? Before admitting defeat, I'd like to describe an older attempt to find a rate of time's passage. At first sight it will seem to richly merit the neglect it has received, but I hope to use it as a stepping-stone to a view that is worth some consideration.

In his 1980 book, *Aspects of Time*, George Schlesinger argues (chapter II, section 4) that the infinite regress of time dimensions that seems to doom the rate of one second per second can be stopped at the second level. His idea is very simple. If the rate of ordinary time can be given as a ratio of intervals in the first time dimension to that of a second, distinct time dimension (the rate of time's passage being one second per SECOND), then the rate of passage in the second time dimension can be given as a ratio of intervals in it to intervals in the first time dimension (say, SECONDS per second). There is no need to hypothesize an infinite hierarchy of time dimensions to give a rate of time's passage. Two suffice.

Even two is too much, though.[9] The ever-vigilant Nathan Oaklander (1983, 391) says that Schlesinger's "two-dimensional time hypothesis… is as beset with difficulties as the conception of temporal becoming it is supposed to render intelligible." This claim is, if anything, an understatement. It is difficult to state in a precise yet plausible way the relation between events in these two time series,[10] but the underlying problem is that the hypothesis of a second time dimension attempts to explain the philosophically

[9] For a more positive assessment of the possibility of multiple time dimensions, see Craig and Weinstein (2009).

[10] Especially if one works, like Oaklander, within the standard McTaggartian framework. I refer the reader to his paper for details.

puzzling notion of temporal becoming in the first time dimension by hypostatizing a second version of that same notion. Duplicating the problem does not diminish it.

2. Rates and Relativity

But hold on a minute. We've been following the custom of the literature on this problem by talking about time without making any reference to spacetime structure. We have, then, been implicitly taking spacetime structure to be that of common sense and pre-relativistic physics, implicitly assuming that times are successively occurring global hyperplanes of simultaneous events. Might not the problem look different from, say, the perspective of the special theory of relativity? It well might, as I soon hope to demonstrate. I will try to show how one can retain the advantage of two time dimensions without the oddity of actually postulating them. Minkowski spacetime $\mathcal{M}$, after all, has only one time dimension.

To make our presentation simpler, let us suppose that $\mathcal{M}$ has also only one spatial dimension as well. In this ((1+1)-dimensional) Minkowski spacetime $\mathcal{M}$ let us choose an inertial frame $\mathcal{F}$ and two points that are timelike separated. For our purpose, this latter phrase means that the worldline of some unaccelerated object, like a clock, can intersect or coincide with both (point) events. The clock, of course, need not be at rest in the frame $\mathcal{F}$, and we will in fact suppose that it moves with some constant velocity *v* in $\mathcal{F}$. Finally, we will suppose that (in $\mathcal{F}$ of course) the spatial distance between the two timelike separated points is D, while the time difference between them is T. [11]

If a clock is able to be present at both events, then the distance between them must be such that an object traveling at less than light speed can move from one to the other. That is, we know that D < cT, where c is is the speed of light. Put another way,

[11] The following discussion is adapted from Mermin (2005, chapter 8), in which he presents "an entertaining consequence of the invariance of the interval." (86)

$$D/T \equiv v < c$$

where v is the velocity of the clock in $\mathcal{F}$. It follows (in an elementary way in the special theory of relativity) that the time that is indicated by the clock between the earlier and the later event, if it is an accurate clock, is

$$T_0 = T\sqrt{1 - v^2/c^2}\,.$$

T_0 is the time measured by the clock in its proper frame, the frame in which it is at rest. In that frame, the distance between the two chosen events, D_0, is evidently equal to 0.

A basic fact about Minkowski spacetime is the invariance (between inertial frames) of the spacetime interval between two points. We can write the interval as $T^2 - D^2/c^2$. The invariance of the interval tells us that

$$(T_0)^2 - (D_0)^2/c^2 = (T_0)^2 = T^2 - D^2/c^2.$$

If we divide through by T^2 and rearrange, we have

$$(T_0/T)^2 + D^2/c^2T^2 = 1$$

or

(1) $$(T_0/T)^2 + v^2 = 1.$$

if we use units in which the speed of light c is equal to 1.

If time does indeed pass, then the proper time read by an ideal clock is probably as direct a representation of it as one can find in physics. And what is the rate at which this time passes? If the clock is at rest--that is, if $v = 0$--then proper time passes at just the rate at which time passes in the original, chosen inertial frame, since $T_0/T = 1$. In a frame in which the clock is moving, the rate at which proper time passes slows with respect to coordinate time T in such a way that sum of $(T_0/T)^2 + v^2$ remains constant.

N. David Mermin (2005, 87) describes this situation this way:

> Now a stationary clock moves through time at 1 nanosecond [of proper time] per nanosecond [of frame or coordinate time] and does not move through space at all. But if the clock moves through space—i.e. the larger *v* is—the slower it moves through time—i.e. the smaller T_0/T is—in such a way as to maintain the sum of the squares of the two at 1. It is as if the clock is always moving through a union of space and time—spacetime—at the speed of light. If the clock is stationary then the motion is entirely through time (at a speed of 1 nanosecond per nanosecond). But in order to move through space as well, the clock must sacrifice some of its speed through time…

A bit of algebra will yield the rate of passing of coordinate time in terms of proper time, if one wishes. So here we have a representation of the rate of time's passing, we have varied rates, and there is no invocation of a second time dimension à la Schlesinger, although there are two kinds of time. I conclude that whatever difficulties there are in understanding time in Minkowski spacetime, the problem of ascertaining the rate of time's passing is not one of them.

3. Reformulations and Replies

What do I mean when I say that there are two kinds of time rather than two temporal dimensions in my suggestion above?

The first kind of time that I have in mind is *coordinate time, t.* An idealized clock is postulated--a clock, that is, whose periods are completely regular--and supposed to be at rest. A point is marked on the worldline of that clock as time zero and a time unit is chosen. Then other similar idealized clocks placed at varying distances from the original clock are synchronized with it using light signals. In this fashion a grid of synchronized clocks extending arbitrarily far through space can be set up.[12]

The second kind of time, *proper time* τ_{ab}, between two timelike separated events a and b along a given timelike path or worldline is defined as

$$\tau_{ab} = \int_a^b [dt^2 - (dx^2/c^2)]^{1/2} = \int_a^b d\tau,$$

in our (1+1)-dimensional spacetime. The proper time difference between two points depends on the path that connects them but is independent of inertial frame, since

$$d\tau^2 \equiv -ds^2/c^2,$$

where

$$ds^2 = dx^2 - c^2dt^2$$

is the infinitesimal frame-invariant Minkowski metric. On the other hand, coordinate time varies with inertial frame according to the Lorentz transformations but is not dependent on any worldline connecting the two points in the way that proper time is.

There are two further important points to note. First, when a clock is at rest in some inertial frame, then $dx = 0$ and so $d\tau = dt$. It follows that the coordinate time interval between two events on its worldline is equal to the proper time interval between them. For this reason proper time is often conflated with coordinate time, but as emphasized above, the two are quite distinct.[13]

Second, the curves connecting two events along which proper time is marked out need not be inertial (or straight) lines. Proper time is defined along the curved paths

[12] For more details of this construction, nearly any standard book on the special theory of relativity will do. See, for instance, §4 of chapter 1 of Taylor and Wheeler (1963).

[13] For a more detailed discussion of this matter see Arthur (2008, §5).

of accelerating, as well as inertially moving, bodies.[14] In light of this fact, it is usual to add the idealization that clocks following such paths in spacetime are not disrupted by the accelerations they encounter (unlike a normal clock, for instance, which may cease to function if it is dropped on a hard floor.) The assertion that such ideal clocks (regular periods, impervious to accelerations) keep proper time is called the *Clock Hypothesis* (CH).[15]

With these facts in mind, it might be useful to discuss a plausible objection to my account of the rate of time's passing. Is it possible to develop an argument with regard to space that would be parallel, or at least analogous to, the argument I gave above that one could find in Minkowski spacetime a way to indicate a (variable) rate of the the passing of time? If so, would that not trivialize my argument?

Let us then begin with two events or points in spacetime that are spacelike separated and choose some arbitrary frame $\mathcal{F}$ in which their spatial separation is D and their temporal separation is T.[16] In this case, D > cT.

There is then a unique inertial frame, $\mathcal{F}_0$, in which the two specified events are simultaneous. We can further suppose that a thin rod connects these two events and that the rod is at rest in $\mathcal{F}_0$.

If D_0 is the length of the rod in $\mathcal{F}_0$, then we know that

(2) $$D = D_0\sqrt{1 - {v^2}/{c^2}}$$

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[14] "Occasionally one encounters the misconception that special relativity can deal only with motion at constant velocity. Nothing could be further from the truth." (Hartle 2003, page 62, note 9)

[15] For a perceptive discussion of this neglected aspect of relativity theory see Arthur (2010).

[16] We will revisit this assumption shortly.

is the length of the rod in $\mathcal{F}$, if $\mathcal{F}$ moves with velocity *v* with respect to $\mathcal{F}_0$,.

And as before, the invariance of the spacetime interval seems to tell us that

(3) $$(T_0)^2 - (D_0)^2/c^2 = - (D_0)^2 = T^2 - D^2/c^2,$$

since $T_0 = 0$. Then of course

$$T^2 + (D_0)^2 = D^2,$$

if we use units in which $c = 1$, and

$$(D_0)^2/D^2 + T^2/D^2 = 1,$$

and it is tempting to think that one can conclude, in imitation of the argument above, that

(4) $$(D_0/D)^2 + 1/v^2 = 1.$$

Were this so, it might be thought that (4) underwrites an argument for the variable passing of space just as I claim that (1) did for the passing of time.

But it is not so. Formula (3) does not correctly reflect the invariance of the interval because the points or events that are connected by the distances in the two frames are not the same pairs of points. Formula (2) is the transformation formula for the length of the rod that we supposed connected our two originally chosen spacelike separated points. In $\mathcal{F}_0$ these two points are simultaneous and the distance between them represents the length of the rod. These two points will not be simultaneous in $\mathcal{F}$ and so the quantity D, the length of the rod in $\mathcal{F}$, must represent the distance between a *different* pair of points, a pair of points simultaneous in $\mathcal{F}$.[17] The spacetime interval is the same in different frames only for the *same* pair of points.

However, it is possible not to make this mistake. One can choose two distinct spacelike separated points in $\mathcal{M}$ and derive the following formula:[18]

(5) $$\left(\frac{D_0}{D}\right)^2 + v^2 = 1,$$

where D_0 is the distance between the two points in a frame $\mathcal{F}_o$ in which they are simultaneous and D is the distance between *the same two points* in a frame $\mathcal{F}$ moving with velocity v ($0 \leq v < 1$) with respect to $\mathcal{F}_o$.

The distances D_0 and D no longer represent lengths of objects, given our usual way of ascertaining lengths, but can one construe (5) as telling us that the rate of passing of proper space increases with respect to the passing of coordinate space as the relative velocity of $\mathcal{F}$ increases with respect to $\mathcal{F}_0$? I claim that it merely tells us the ratio of a chosen length measured in two inertial frames.

Am I simply opting for a metaphysically inflated reading of (1) while insisting on metaphysically deflated reading of (5)? Perhaps, but I have a reason for my asymmetric attitude. The clock hypothesis tells us that ideal clocks indicate proper time and do so accurately despite any acceleration. That is, they mark (proper) time equably without regard to anything external. If time passes, then what they indicate is the amount of passed time, albeit only along a given path connecting the two points. When two spacelike points are kept fixed, as in (5), however, the distance D between them no longer represents the length of a measuring rod, as we have seen. I think it then follows that no supposedly analogous argument that measuring rods represent the passing of space is really analogous to the argument developed from (1).

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[17] Though of course one of the points can be in both pairs.

[18] As John Manchak pointed out to me.

Let me remind the reader, as a final step, of the structure of my argument. I took as a working hypothesis the vague idea that time passes and then showed that, given this hypothesis, some sense could be made of the rate of such passage in Minkowski spacetime. Thus one objection to the working hypothesis is removed. While there is much in our experience to support this working hypothesis, independent of the argument given here, there is no such support for the hypothesis that space passes. Even if a formula like (5) could possibly remove an objection to it, it there is no independent ground for the hypothesis itself.

My conclusion is that in one interesting spacetime, Minkowski spacetime, the old objection that it is not possible to specify a rate of time's passing fails. My argument should not leave one wondering whether space too passes, but it might well leave one with the conviction that proper time and the clock hypothesis are well worth further consideration, as in Arthur (2010).

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